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Fermionic excitation in quark matter

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Abstract: We discuss the fermionic excitation spectra of the dense QCD and QED plasma at zero temperature, and analyze the stability of the normal (perturbative) ground state. The standard re-summed perturbation theory breaks down because the infrared (IR) divergences show up in the spectra near the Fermi surface. We employ the effective field theory approach and the renormalization group to investigate the dynamics near the Fermi surface. Our results indicate that the dense QCD and QED plasma are non-Fermi-liquids.

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1. Introduction

There are several situations where hot and/or dense deconfined quark-gluon plasma (QCD plasma) might be formed; e.g., the interior of the neutron stars, the early universe, and the relativistic heavy-ion collisions. It is commonly accepted that for the QCD plasma the free gas description gives a useful starting approximation. The another example for which such treatments are frequently used is the QED plasma (electron gas). This corresponds to the following assumption: for sufficiently hot and/or dense case, perturbation theory (PT) along with some appropriate resummation procedure (re-summed perturbation theory, rsPT) gives a successful scheme to describe the dynamics.

In this talk we concentrate on the dense case, by which we mean the high-density but zero-temperature case. For this case, the above assumption corresponds to that the ground state is the normal Fermi-liquid state. We discuss the fermionic excitation spectra in the dense plasma, and examine the validity of the assumption. We emphasize that the physics of the dense QCD and QED plasma is profoundly connected with the IR behavior of the theory.

2. Perturbation theory calculation of the ground-state energy

A peculiar property of QCD is the asymptotic freedom, which is a basis of successful applications of perturbative methods to high-energy processes[1]. Naively, one would expect that the asymptotic freedom would support PT starting from the free quark gas. Such a picture was pushed forward by Collins and Perry[2]. They employed a renormalization group (RG) argument similar to those in the high-energy process. The following behavior of the thermodynamic quantities, when the Fermi momentum p_F is scaled up by κ , was obtained:

$$X(\kappa p_F, g, m, \mu) = \kappa^{D_X} X(p_F, \bar{g}(\kappa), \bar{m}(\kappa), \mu), \quad (1)$$

where X stands for some thermodynamic quantity like the energy density, and D_X is its mass dimension. g and m are the gauge coupling constant and the quark mass renormalized at the scale μ , while $\bar{g}(\kappa)$ and $\bar{m}(\kappa)$ are the corresponding running quantities. Because $\bar{g}(\kappa) \rightarrow 0$ as $\kappa \rightarrow \infty$, (1) seems to support the naive expectation discussed above.

There is a subtle point in this sort of RG arguments: the difficulties might arise from the IR divergences, because the gluons are massless and also $\bar{m}(\kappa) \rightarrow 0$ as $\kappa \rightarrow \infty$. In the medium, however, the gluons acquire the Debye screening mass and the plasma mass due to the interaction with the quark gas[3]. Collins and Perry argued that these “effective masses” would act as an IR cut-off.

Following this line, and in order to demonstrate explicitly IR safety of PT at high density, the calculation of the ground-state energy, up to and including the effects of $O(\alpha^2)$ ($\alpha = g^2/4\pi$), was carried out in [4, 5]. At $O(\alpha^2)$, the diagrams, which involve the 3-, 4-point gluon couplings or the ghost-gluon coupling, contribute. As for the IR divergences, their results can be summarized as follows:² the second order ($O(\alpha)$) graph is free of the IR divergences. At the fourth order $O(\alpha^2)$, the individual diagrams contain the IR divergences, but most of them cancel out between the different diagrams. The only one exception is the diagram involving the two fermion bubbles; thus the order by order PT is suffered from the IR divergences. Fortunately, however, the resummation of the higher order graphs of the RPA-type (ring

²The ultraviolet (UV) divergences can be completely removed by the counter terms determined at $p_F = 0$.

diagrams) renders the result finite. This is similar to the well-known nonrelativistic (NR) electron gas calculation[6]. The resummation of the RPA produces a finite static screening mass to the “dangerous modes” of the gluons, and the difficulty can be avoided. The rsPT for the dense QCD plasma is IR safe in accord with the conjecture of ref.[2], at least for the computation of the ground-state energy.

There is one price due to the resummation: The re-summed RPA series (the sum of the ring diagrams involving two or more fermion bubbles) give the contribution of $O(\alpha^2 \log \alpha)$, which are dominant compared with the other $O(\alpha^2)$ graphs.

3. Fermionic excitation spectra

In this section we compute the quark excitation spectra in the dense QCD plasma in the framework of rsPT discussed in sect.2.

The excitation energy of the quarks in the medium can be defined as the change in the total energy when a quark is added (removed) into (from) the system. This can be expressed as a functional derivative of the energy density \mathcal{E} with respect to the Fermi distribution function $n(p)$:

$$\epsilon(p) = \frac{\delta \mathcal{E}}{\delta n(p)}, \quad (2)$$

$\epsilon(p)$ is nothing but the quasiparticle energy in the Landau Fermi-liquid theory[7]. In PT, the (free) quark propagator in the rest frame of the $T = 0$ plasma is[3]

$$S(p) = \frac{1}{\not{p} - m + i\varepsilon} + i\pi \frac{\not{p} + m}{E(p)} \delta(p^0 - E(p)) n(p) \equiv S_F(p) + S_D(p), \quad (3)$$

with $E(p) = \sqrt{\mathbf{p}^2 + m^2}$ and $n(p) = \theta(p_F - |\mathbf{p}|)$. Because S_D involves $n(p)$, the functional derivative (2) cuts the one quark-line in the diagrams for the energy density. Including the contributions up to $O(\alpha^2 \log \alpha)$ for \mathcal{E} , we obtain as the corrections to the free quark spectrum $E(p)$:

$$\epsilon_1(p) = \frac{m}{E(p)} \bar{u}(p) \Sigma(p) u(p). \quad (4)$$

$u(p)$ is the free quark spinor normalized as $\bar{u}u = 1$. $\Sigma(p)$ is the RPA-type self-energy given by the diagrams of fig.1. For the computation of $\epsilon_1(p)$, we perform the Wick rotation for the gluon loop momentum integration. The procedure is similar to the one discussed in [8, 9]. One obtains the two types of contributions, depending on whether the intermediate quark line connecting the external legs in the diagrams of fig.1 is off-shell or on-shell. We denote the off-shell part by $\epsilon_1^W(p)$ while the on-shell part by $\epsilon_1^C(p)$, as $\epsilon_1(p) = \epsilon_1^W(p) + \epsilon_1^C(p)$. ϵ_1^C can be compactly expressed as

$$\epsilon_1^C(p) = 2N_f N_c \int_p^{p_F} \frac{dq q^2}{(2\pi)^2} \int_{-1}^1 d \cos \theta f(\mathbf{p}, \mathbf{q}). \quad (5)$$

$\cos \theta = \mathbf{p} \cdot \mathbf{q} / pq$, and $f(\mathbf{p}, \mathbf{q})$ is the exchange-type interaction (averaged over spin and color) between the two quarks with the momenta \mathbf{p}, \mathbf{q} . Here and in the following, we consider the case of $SU(N_c)$ color and the N_f -light quarks.

ϵ_1^W is given as a (Wick-rotated) 4-dimensional euclidean integral. It can be shown, by explicit computation, that ϵ_1^W is free of the IR divergences[10]. On the other hand, ϵ_1^C is IR

divergent! f assumes the following form[8, 9]:

$$f(\mathbf{p}, \mathbf{q}) = \sum_{X=L,T} \frac{A_X}{(p-q)^2 - \Pi_X(p-q)}, \quad (6)$$

where “L” and “T” denote the longitudinal and the transverse modes of the gluons. They are the two independent gluonic modes in the medium. Π_X is the self-energy for these modes due to the RPA resummation. It is given by the gluon self-energy $\Pi_{\mu\nu}$ as $\Pi_L(k) = \Pi_{11}(k) - \Pi_{00}(k)$, $\Pi_T(k) = \Pi_{33}(k)$ if we choose \mathbf{k} to lie on the x -axis. A_X is the dimensionless function accounting for the vertex structure of the quark-gluon coupling and is proportional to α . The IR divergence shows up near the Fermi surface. For $p \cong p_F$, we obtain

$$\epsilon_1^C(p) \cong (p - p_F) \frac{N_f N_c}{4\pi^2} \sum_{X=L,T} \int_{-1}^1 d\cos\theta \frac{A_X}{1 - \cos\theta + \hat{\Pi}_X(p-q)}, \quad (7)$$

where the integrand is evaluated at the Fermi surface ($|\mathbf{p}| = |\mathbf{q}| = p_F$), and $\hat{\Pi}_X(k) = (\Pi_X(k)/2p_F^2)_{k^0=0}$. The integral would be logarithmically divergent if $\hat{\Pi}_X = O(\theta^2)$ as $\theta \rightarrow 0$. This is the IR divergence due to the soft gluons with the momentum $p_\mu - q_\mu \rightarrow 0$.

Now the key quantity which determine the IR behavior of the fermionic excitation spectra is the so-called “screening mass” of the gluons defined by

$$M_X^2 = \lim_{k \rightarrow 0} \lim_{k_0 \rightarrow 0} \Pi_X(\mathbf{k}, k_0). \quad (8)$$

In the RPA resummation scheme, Π_X contains the quark one-loop diagrams. One finds:

$$M_L^2 = 2N_f \frac{\alpha}{\pi} p_F^2; \quad M_T^2 = 0. \quad (9)$$

These are the gauge invariant results. Therefore, the L-modes give the IR safe result, while the T-modes cause the IR divergences. The screening of the L-modes corresponds to the Coulomb screening in the NR electron gas[6]. The non-screening of the T-modes (magnetic modes) is a result of gauge invariance $k_\mu \Pi^{\mu\nu} = 0$ in the framework of PT.

We now find for the quark excitation spectra:

$$\epsilon(p) = \epsilon_F + (p - p_F)v + O((p - p_F)^2), \quad (10)$$

where ϵ_F is the Fermi energy, and

$$v = v_0 - C_F \frac{\alpha(p_F)}{\pi} v_0^2 \left[\log \left| \frac{p_F}{p - p_F} \right| + O(1) \right], \quad (11)$$

with $v_0 = p_F/E(p_F)$, $C_F = (N_c^2 - 1)/2N_c$. $\alpha(p_F)$ is the running coupling constant at the scale p_F . The result indicates the logarithmic singularity at $p = p_F$, and thus the negatively infinite slope of the spectra at the Fermi surface ($v_F = \partial\epsilon(p)/\partial p|_{p=p_F} \rightarrow -\infty$). The behavior of the spectra is shown in fig.2. Several comments are in order here:

(a) Also for the case of the hot QCD plasma (at $p_F = 0$), the T-modes of the gluons do not get screened, at least up to $O(\alpha)$ [3]: it was pointed out[11] that this causes the breakdown of PT for the thermodynamic potential at the sixth order. On the other hand, for the case

of the dense plasma, there has been no clear indication of such difficulties: all energy density computations by rsPT have been worked out successfully as discussed in sect.2. Our results reveal that the RPA breaks down if one computes the excitation spectra.

(b) In the present treatment, the non-abelian character of the theory does not enter explicitly, except for the running coupling constant $\alpha(p_F)$ and the color factors of (11). Thus the result can be easily translated into the case of an abelian gauge theory, i.e., into the QED plasma. This indicates that a naive extension of the famous work on the NR electron gas by Gell-Mann and Brueckner[6] to the relativistic case breaks down:³ in the NR case, the quasiparticle energy in the RPA scheme was obtained successfully by Gell-Mann[13]. Our results (10),(11) do not reduce to Gell-Mann’s result even if we go to the NR limit ($p_F/m \rightarrow 0$). The discrepancy comes from the IR divergent contributions discussed above. Our results confirm the argument of [14] by employing the full relativistic framework.

(c) The breakdown of rsPT poses a question on the usefulness of the free gas description of the dense QCD and QED plasma. The spectra with the inversion may indicate the instability of the normal (Fermi-liquid) state (see also [14, 15, 16, 9]).

The breakdown of the RPA as rsPT for high density has a great impact. Before drawing physical implications, however, we have to investigate the effects beyond the RPA, and examine whether those effects cancel the above IR divergences or not. In the next section we employ the RG approach and try to go beyond the RPA.

4. Renormalization group approach

In this section we investigate the ground state of the dense QCD plasma in the framework of the renormalization group (RG) à la Wilson. The Wilson RG is a useful tool to analyze the low energy dynamics of the system and to construct an effective field theory. It is especially powerful when the low energy dynamics is dominated by gapless (massless) excitations. In this case, the effective field theory corresponds to an IR fixed point of the RG transformation. A well-known example is the application to critical phenomena[17].

Recently, there has been progress in the applications of the RG to (NR) many-fermion systems[18, 19, 20, 21]. The key observation is that the system of many-fermions in the normal state is dominated by gapless excitations at low energy, due to particles and holes near the Fermi surface. The Landau Fermi-liquid theory corresponds to the Gaussian (mean-field) fixed point. This view point also opens a possibility to classify the possible phases of the system by a few number of “relevant perturbations” which would lead to a RG flow toward another IR stable fixed point rather than the Fermi-liquid fixed point. For example, it has been shown[18, 19, 20] that, for the case of 3-dimensional NR Fermi systems with rotational invariance, only the pairing interactions leading to the Cooper pairs could be (marginally) relevant; otherwise, the system is described as a normal Fermi-liquid.

Now we apply the RG approach to the dense QCD plasma, and investigate whether the Fermi-liquid fixed point is a IR stable one or not. If it were not stable, the assumption discussed in sect.1 would be wrong. As the first step, we have to write down the low energy effective action S_Λ . It is obtained by integrating out the “fast modes”, which have the momenta greater than the UV cut-off Λ , in the original action. The modes within Λ , which we call the “slow modes”, constitute our “physical” degrees of freedom dominating the low

³The ground-state energy of the relativistic electron gas was obtained successfully in[12, 4, 5]

energy dynamics, and S_Λ should be expressed by them. One important point is that all possible relevant or marginal interactions (perturbations) between the slow modes, except those forbidden by symmetry, should be included in S_Λ . S_Λ for our RG analysis is:

$$\begin{aligned}
S_\Lambda = & \int_{|p-p_F|<\Lambda} d^4p \bar{\psi}(p) \gamma^0 [p^0 - \epsilon_F - v_F(p - p_F)] \psi(p) + \int_{|q|<\Lambda} d^4q q^2 (\tilde{A}_i^a(q))^2 \\
& + \int_{|p'-p_F|<\Lambda} d^4p' \int_{|p-p_F|<\Lambda} d^4p \int_{|q|<\Lambda} d^4q (2\pi)^4 \delta^{(4)}(p' - p - q) g \bar{\psi}(p') \gamma^i t^a \psi(p) \tilde{A}_i^a(q) \\
& + (4\text{-fermion interaction terms}) + (\text{gluon interaction terms}).
\end{aligned} \tag{12}$$

Here the quark (ψ) and the gluon (\tilde{A}_i^a) fields are in the momentum space.⁴ The momentum integrations are restricted within Λ and involve the slow modes only. Note that, for the fermions, the slow modes are those near the Fermi surface. Their excitation spectra are linearized as $\epsilon(p) \cong \epsilon_F + v_F(p - p_F)$. \tilde{A}_i^a stand for the T-modes of the gluons. The L-modes of the gluons do not appear in (12): as we saw in sect.3, the “mass” of the gluons relevant for the fermionic excitation spectra is the screening mass. The L-modes become massive due to the Debye screening, and thus decouple from the low energy dynamics. On the other hand, the screening mass for the T-modes is forbidden by gauge invariance. The integration of the L-modes and other fast modes generate the 4-fermion interactions, but only a few types of them, which satisfies special kinematical conditions, could be marginal (at the tree level). The “gluon interaction terms” may involve the 3- and 4-point couplings between the gluons, and also the gauge terms (the gauge fixing terms and the Faddeev-Popov ghost terms), but the explicit forms of them are irrelevant for the following discussion.

As a result of mode elimination, the coupling parameters (v_F , g , the 4-fermion couplings, etc.) depend on Λ , and thus are the running couplings in the RG. The partition function is given as the functional integral by the slow modes

$$Z = \int [d\psi d\bar{\psi} d\tilde{A}] e^{iS_\Lambda}. \tag{13}$$

We perform the RG transformation and derive the RG equations for the running couplings. We divide our physical space further into the slow modes ($|p - p_F|, q < \Lambda/s$, $s > 1$) and the fast modes ($\Lambda/s < |p - p_F|, q < \Lambda$), and integrate the fast modes out in (13). The scale transformation $|p - p_F|, q \rightarrow |p - p_F|/s, q/s$ recovers the phase space reduced by mode elimination. The comparison of the new effective action with (12) gives a flow of the coupling parameters when Λ is scaled down. In order to trace the RG flow toward the Fermi-liquid fixed point, we perform the scale transformation to the quark and the gluon fields as $\psi \rightarrow s^{3/2} \sqrt{Z_2} \psi$; $\tilde{A}_i^a \rightarrow s^3 \sqrt{Z_3} \tilde{A}_i^a$, where Z_2 and Z_3 are the renormalization constants for these fields.

The Fermi velocity v_F determines the behavior of the fermionic excitation spectra near the Fermi surface. This is a marginal coupling at the tree level. By performing mode elimination at the 1-loop level (see fig.3), we obtain the RG equation:

$$\Lambda \frac{dv_F}{d\Lambda} = C_F \frac{\alpha}{\pi} v_F^2, \tag{14}$$

⁴These fields stand for the fermionic and the bosonic elementary excitations with the same quantum numbers as the quark and the gluon; we call them simply by the quark and the gluon in this section.

with $\alpha = g^2/4\pi$. This should be combined with the β -function, $\beta = \Lambda(d\alpha/d\Lambda)$. As a boundary condition, we match our effective field theory with the QCD for the zero density, at the initial (high-energy) scale $\Lambda_0(\sim p_F)$ of the successive RG transformation toward the Fermi surface: $\alpha(\Lambda) = 1/[\beta_0 \log(\Lambda/\Lambda_{\text{QCD}})]$, or $\beta = -\beta_0\alpha(\Lambda) + O(\alpha^2)$, for $\Lambda \geq \Lambda_0$. ($\beta_0 = (11N_c - 2N_f)/6\pi$ and Λ_{QCD} is the QCD scale parameter.) This implies, by the continuity, $\alpha(\Lambda)$ grows at the beginning if we scale down Λ from Λ_0 . When we go to the lower energy, we have the two possibilities, depending on the behavior of $\beta(\alpha)$ for $\alpha \geq \alpha(\Lambda_0)$:

- (i) $\beta(\alpha) < 0$ (i.e., no zero of $\beta(\alpha)$) for $\alpha \geq \alpha(\Lambda_0)$.
- (ii) $\beta(\alpha^*) = 0$ for some $\alpha^* \geq \alpha(\Lambda_0)$.

The case (i) corresponds to the case where α is a marginally relevant coupling and continues to grow at low energy. In this case, the low energy effective theory is a strong coupling theory and thus does not correspond to the Fermi-liquid fixed point.

The case (ii) corresponds to the case where α^* gives an IR fixed point. If α^* is small, this may correspond to the Fermi-liquid fixed point, and thus needs more detailed analysis. In this case, (14) can be easily integrated to give

$$v_F(\Lambda) = \frac{v_F(\Lambda_0)}{1 + C_F(\alpha^*/\pi) v_F(\Lambda_0) \log(\Lambda_0/\Lambda)}. \quad (15)$$

Here $v_F(\Lambda_0) > 0$. The result implies $v_F \rightarrow 0$ when one goes to the low energy limit $\Lambda \rightarrow 0$, i.e., near the Fermi surface. Note that if we expand (15) perturbatively and retain the leading term, we recover the previous result (11). This indicates that the discussion of sect.3 corresponds to assuming the case (ii); the RG makes it possible to sum up the leading logarithms in all orders of PT.

One might say that the result is “safe” if the case (ii) is realized. We have no IR difficulty. (15) indicates that v_F is a marginally irrelevant coupling at low energy. So, the low energy effective theory would be given by the Fermi-liquid fixed point with $\alpha = \alpha^*$ and $v_F = 0$. However, we argue that this fixed point will never be stable: first, because $\alpha^* \neq 0$, there exist the long-range attractive interaction between the quarks due to the exchange of the T-modes of the gluons. Second, the density of states at the Fermi surface $N(0) \propto 1/v_F$ is infinitely large (see fig.2).⁵ The infinite density of states will enhance the attractive channels of the 4-fermion interactions, and will drive the instability of the Fermi-liquid fixed point. For example, the BCS-type pairing interaction V gets the 1-loop correction $\sim -N(0)V^2$, and is strongly enhanced at low energy for $V < 0$. As an origin of the attractive V , we need not other degrees of freedom like the phonon in the usual superconductors; it could be generated solely through mode elimination in gauge theory, e.g., by the Kohn-Luttinger effect[22].

Our RG analysis indicates the two possibilities for the effective field theory for the dense QCD plasma: (i) a strong coupling theory with the large α ; (ii) a “would-be fixed point” with $\alpha = \alpha^*$, $v_F = 0$. We conclude that the ground state of dense QCD plasma is not the normal Fermi-liquid state for both cases.

In order to determine which case is realized, we have to compute the β -function from the relevant 1-loop diagrams. For the simpler case of the dense QED plasma, we can show it corresponds to (ii) at 1-loop[10]. (Note, in this case, $\beta_0 < 0$.) Thus, the results similar

⁵In the NR case where the T-modes of the gauge fields are not included, the running of v_F is not so significant, and v_F is considered to go to a nonzero value for $\Lambda \rightarrow 0$ [18].

to (15) can be obtained: the ground state of the electron gas may be the superconducting state, even without the phonon degrees of freedom.⁶

The behavior $v_F(\Lambda) \rightarrow 0$ ($\Lambda \rightarrow 0$) is similar to that of a marginal Fermi-liquid.⁷ The marginal Fermi-liquid is a successful phenomenological model proposed to explain the puzzling normal state properties of Cu-O high-temperature superconductors[25]. That a proper relativistic treatment of the electron gas might lead to superconductivity of a marginal Fermi-liquid would be an interesting possibility as a model of high-temperature superconductivity.

5. Conclusion

We discussed the fermionic excitation spectra of the dense QCD (and QED) plasma. It is shown that the standard rsPT breaks down because the IR divergences show up in the spectra near the Fermi surface. The divergence is due to the non-screening of the transverse modes of the gauge fields. By employing the effective field theory approach and the RG, we classified the two possibilities for the ground state of the QCD plasma: (i) a strong coupling theory where the nonperturbative effects will be important; (ii) a would-be IR fixed point with $\alpha = \alpha^*$ and $v_F = 0$, which would be unstable against the transition to the superconducting state. We concluded that the $T = 0$ QCD (and QED) plasma is a non-Fermi-liquid.

We stress that the physics of the dense QCD and QED plasma is not so simple on the contrary to the usual picture. Our results indicate the importance of the “soft degrees of freedom”; they could appear in the higher order terms in PT as well as in the nonperturbative effects. More efforts are needed to reveal roles of the soft degrees of freedom and to understand the dynamics of the QCD and QED plasma.

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⁶The possibility that the ground state of the relativistic electron gas would be the superconducting state was discussed by Chu, Huang and Polonyi by a different argument[23].

⁷The $v_F \rightarrow 0$ behavior has been also found in a 2-dimensional Fermi system with a Chern-Simons gauge field by Nayak and Wilczek[24].

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Figure Captions

Fig. 1 The self-energy diagrams for the quarks in the RPA (full lines: quarks; wavy lines: gluons).

Fig. 2 The fermionic excitation spectra (dotted line: $v_F > 0$; dashed line: $v_F = -\infty$; full line: $v_F = 0$).

Fig. 3 The 1-loop diagrams which give the RG equation (14). The first diagram is due to the transverse gluon exchange, while the second one is due to the 4-fermion interactions.